Problem Solution

The steps followed to solve the problems are as follows:

1- Determine the UTC (Coordinated Universal Time) for the three observations to calculate the Local Sidereal Time for each observation.
   Nairobi is located at UTC + 3 so the time in UTC is the time given − 3.

2- Calculate the Local Sidereal Time for each observation.

Date	Local Time*	Azimuth Angle (deg.)	Elevation Angle (deg.)	Range (km)	Local Sidereal Time	Universal Time
17-05-20	12:06 AM	272.4822	21.9305	39364	229.322	21:06
17-05-20	1:00 AM	272.4401	22.0888	39348	242.859	22:00
17-05-20	1:30 AM	272.3864	22.5174	39307	250.38	22:30

3- Calculate the three geocentric position vectors corresponding to the three sets of azimuth, elevation and slant range data at each time given the earth observation location:

A) Using the altitude H, latitude ϕ and local sidereal time θ of the site, calculate its geocentric position vector R from the following equation:

$$\mathbf{R} = \left[\frac{R_e}{\sqrt{1 - (2f - f^2)\sin^2 \phi}} + H \right] \cos \phi \left(\cos \theta \hat{\mathbf{I}} + \sin \theta \hat{\mathbf{J}} \right)$$

$$+ \left[\frac{R_e(1 - f)^2}{\sqrt{1 - (2f - f^2)\sin^2 \phi}} + H \right] \sin \phi \hat{\mathbf{K}}$$

B) Calculate the topocentric declination δ using the following equation:

$$\delta = \sin^{-1}(\cos \phi \cos A \cos a + \sin \phi \sin a)$$

C) Calculate the topocentric right ascension α using the following equations:

$$h = \begin{cases} 2\pi - \cos^{-1}\left(\dfrac{\cos\phi\sin a - \sin\phi\cos A\cos a}{\cos\delta} \right) & 0° < A < 180° \\[4mm] \cos^{-1}\left(\dfrac{\cos\phi\sin a - \sin\phi\cos A\cos a}{\cos\delta} \right) & 180° \le A \le 360° \end{cases}$$

$$\alpha = \theta - h$$

D) Calculate the direction cosine unit vec $\hat{\varrho}$ r   using the following equation:

$$\hat{\varrho} = \cos\delta(\cos\alpha\hat{I} + \sin\alpha\hat{J}) + \sin\delta\hat{K}$$

E) Calculate the geocentric position vector r using the following equation:

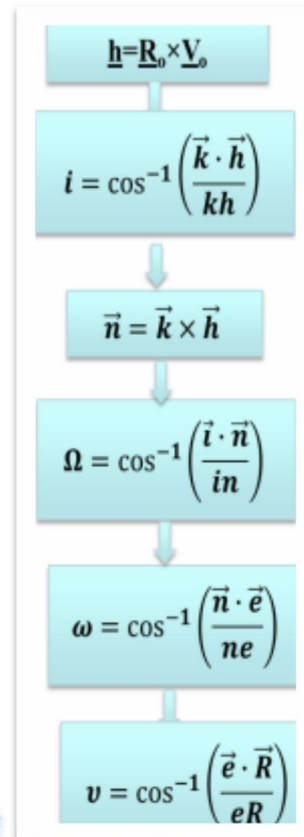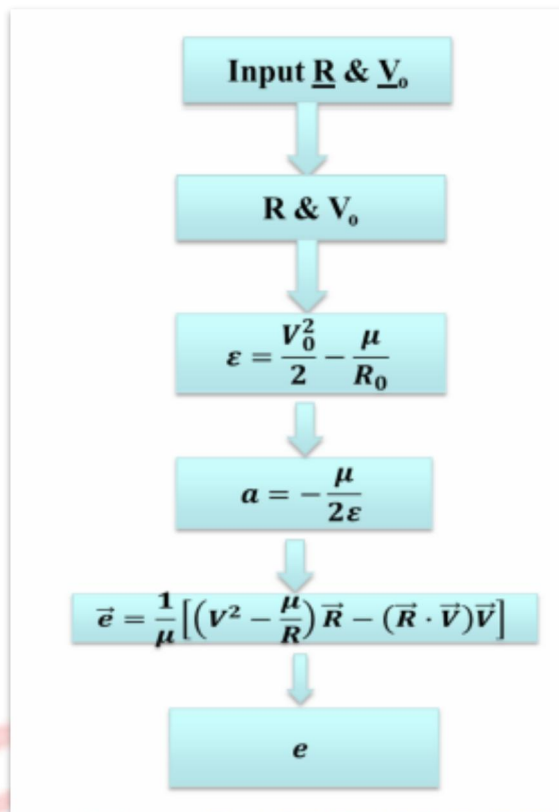$$\mathbf{r} = \mathbf{R} + \varrho\hat{\varrho}$$

The results obtained by MATLAB were as the following:

4- Applying Gibbs' Method to the three obtained position vectors to get the velocity vector corresponding to the second position vector:
   - The result was as following:

We take the initial r and v to be r2 and v2:

5- Knowing r and v the 6 COE could be obtained easily using the below flow graph [1] and the results were:

Input $\underline{R}$ & $\underline{V}_o$

↓

R & V_o

↓

$$\varepsilon = \frac{V_0^2}{2} - \frac{\mu}{R_0}$$

↓

$$a = -\frac{\mu}{2\varepsilon}$$

↓

$$\vec{e} = \frac{1}{\mu}\left[\left(V^2 - \frac{\mu}{R}\right)\vec{R} - (\vec{R}\cdot\vec{V})\vec{V}\right]$$

↓

e

$$\underline{h}=\underline{R}_o\times\underline{V}_o$$

↓

$$i = \cos^{-1}\left(\frac{\vec{k}\cdot\vec{h}}{kh}\right)$$

↓

$$\vec{n} = \vec{k}\times\vec{h}$$

↓

$$\Omega = \cos^{-1}\left(\frac{\vec{i}\cdot\vec{n}}{in}\right)$$

↓

$$\omega = \cos^{-1}\left(\frac{\vec{n}\cdot\vec{e}}{ne}\right)$$

↓

$$v = \cos^{-1}\left(\frac{\vec{e}\cdot\vec{R}}{eR}\right)$$

Since the eccentricity is between 0 and 1, the orbit is **elliptical.**

For elliptical orbit the following equations were applied:

Average angular velocity   Mean Motion (Mean anomaly)

The eccentric anomaly is then connected to the mean anomaly using:

This equation was solved numerically to obtain E with every Mean anomaly using fzero function from MATLAB. It applies newton's method using the following function

$$E_{i+1} = E_i - \frac{E_i - e \sin E_i - M_e}{1 - e \cos E_i}$$

The true anomaly was then obtained by

$$\cos \theta = \frac{e - \cos E}{e \cos E - 1}$$

Then R and V were obtained by:

These were then transformed to the geocentric frame using a transformation matrix.

Circular, parabolic, and hyperbolic orbits are also found in the code but are not of our interest as our orbit is elliptical.

The position and velocity vectors were forced to be calculated at t = 3600 s and it can be found in the last column (not drawn). They are as follows:
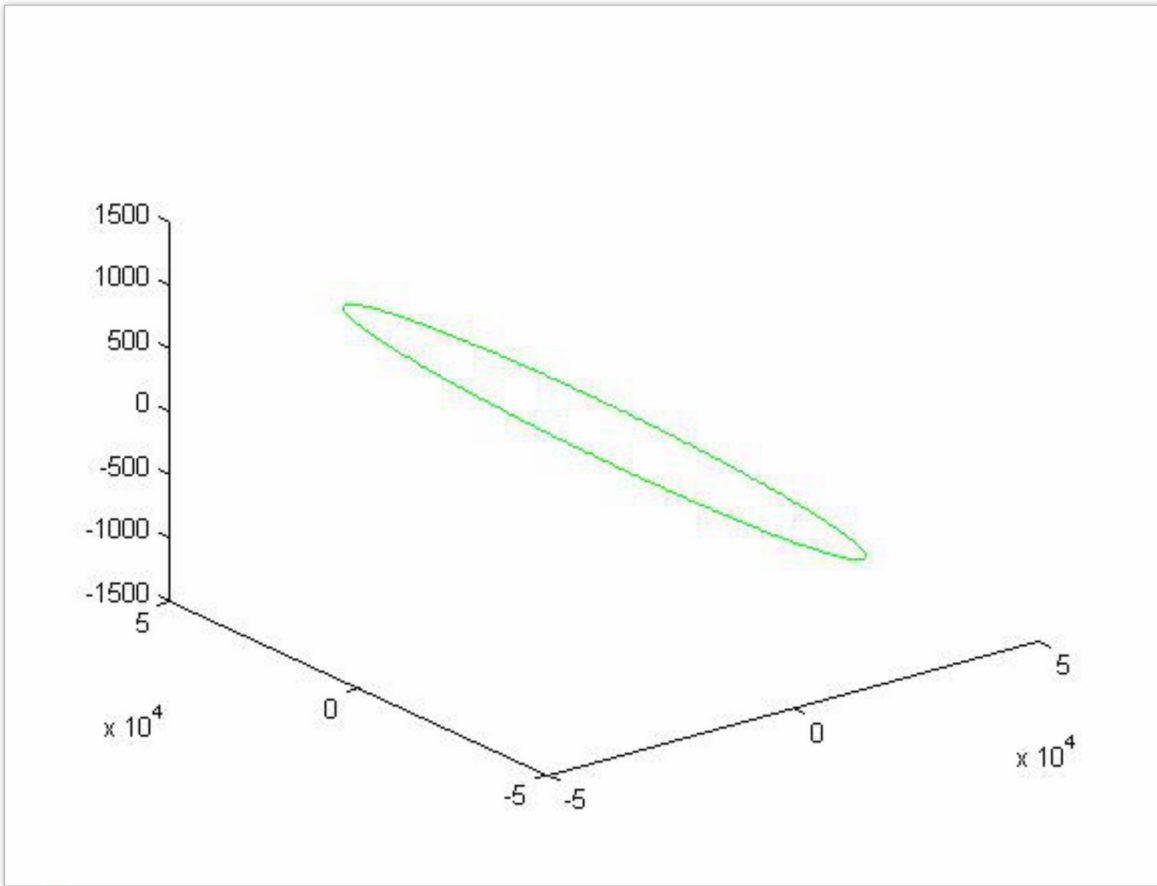
Important Notes:

Only the file 'Main_Orbital.m' needs to be run to solve the problem, it iterates over time and draws the ellipse. It calls the script 'Final_r_v.m' which in turn obtains the initial velocity and position as stated above.

Another method was applied called 'Universal Kepler Equations', it can be found in the MATLAB script 'orbit_solve_draw.m'.

Conclusion

The orbit is an elliptical orbit with a very small eccentricity, (nearly circular). The object is orbiting in a prograde orbit; meaning that it is rotating in an opposite direction to the direction of rotation of Earth. In addition, the object is not expected to escape the orbit or collide with Earth, instead, it will remain in orbit.

<REQU> - required output can be found in red.

References:

1- Orbital Mechanics for Engineering Students
2- Understanding Space: An Introduction to Astronautics 2nd
3- http://www.bogan.ca/orbits/kepler/orbteqtn.html
4- https://www.maplesoft.com/content/EngineeringFundamentals/49/MapleDocument_46/Orbita _Mechanics.pdf