Q1.

$X \sim N_n(0, \sigma^2 I)$

Let $\sigma^2 I = \Sigma$

then the pdf of f(x) =

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then the pdf of x is

$$f(x) = \frac{1}{2\pi^{\frac{n}{2}} |\Sigma|^{\frac{1}{2}}} \exp\left\{ -\frac{1}{2} x^T \Sigma^{-1} x \right\}$$

Here Y=PX

$E(Y) = E(PX) = PE(X) = P.0 = 0$

$Var(Y) = Var(PX) = P^T Var(X) P = P^T \sigma^2 \mathrm{IP} = \sigma^2 P^T P$

Sinse P is orthogonal , $P^T P = I_k$

Hence Var(Y)=$\sigma^2 I_k$

So Y $\sim N_k(0, \sigma^2 I_k)$

Q. 2(a)

$$Q = \frac{1}{3}(Y_1 - Y_2)^2 + (Y_2 - Y_3)^2 + (Y_3 - Y_1)^2$$

$$= \frac{1}{3}(2y_1^{\ 2} + 2y_2^{\ 2} + 2y_3^{\ 2} - 2y_1y_2 - 2y_2y_3 - 2y_3y_1)$$

$$= \frac{1}{3}\begin{pmatrix} y_1 & y_2 & y_3 \end{pmatrix}\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \\ y_2 \end{pmatrix}$$

$$= \begin{pmatrix} y_1 & y_2 & y_3 \end{pmatrix}\begin{pmatrix} 2/3 & -1/3 & -1/3 \\ -1/3 & 2/3 & -1/3 \\ -1/3 & -1/3 & 2/3 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \\ y_2 \end{pmatrix}$$

Results : If X $\sim$ N(0,$\sigma^2 I$)and M is idempotant matrix and than

$$\frac{X^T MX}{\sigma^2} \sim \chi^2(trace(M).$$

Using this result , Say M= $\begin{pmatrix} 2/3 & -1/3 & -1/3 \\ -1/3 & 2/3 & -1/3 \\ -1/3 & -1/3 & 2/3 \end{pmatrix}$

which is idempotant and symmetric matrix and trace(M)=2

$$\frac{Y^T MY}{1^2} \sim \chi^2(trace(M)).$$

$$Y^T MY \sim \chi^2(2).$$

Q. 2(b)

$$Y \sim N_3(0, I_3)$$
$$y \sim iid\ N(0,1)$$
$$\bar{y} = \frac{y_1 + y_2 + y_3}{3} \sim N(0, \frac{1}{3})$$
$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} \sim N_4\left(0, \Sigma_{4X4}\right)$$
$$\Sigma = \begin{pmatrix} 1 & 0 & 0 & 1/3 \\ 0 & 1 & 0 & 1/3 \\ 0 & 0 & 1 & 1/3 \\ 1/3 & 1/3 & 1/3 & 1/3 \end{pmatrix}$$
$$|\Sigma| = 1/9$$
$$Cov(y_1, \bar{y}) = \frac{1}{3} Cov(y_1, y_1 + y_2 + y_3) = 1/3$$
$$similarily\ Cov(y_i, \bar{y}) = 1/3$$

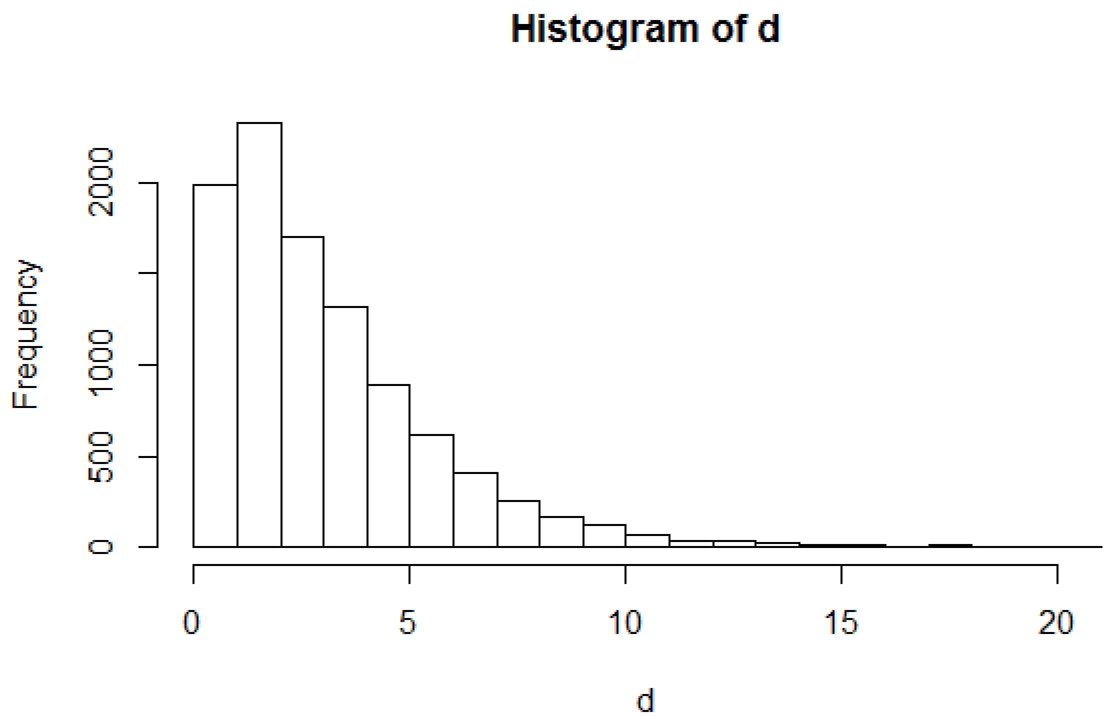
Hence the distribution of V is

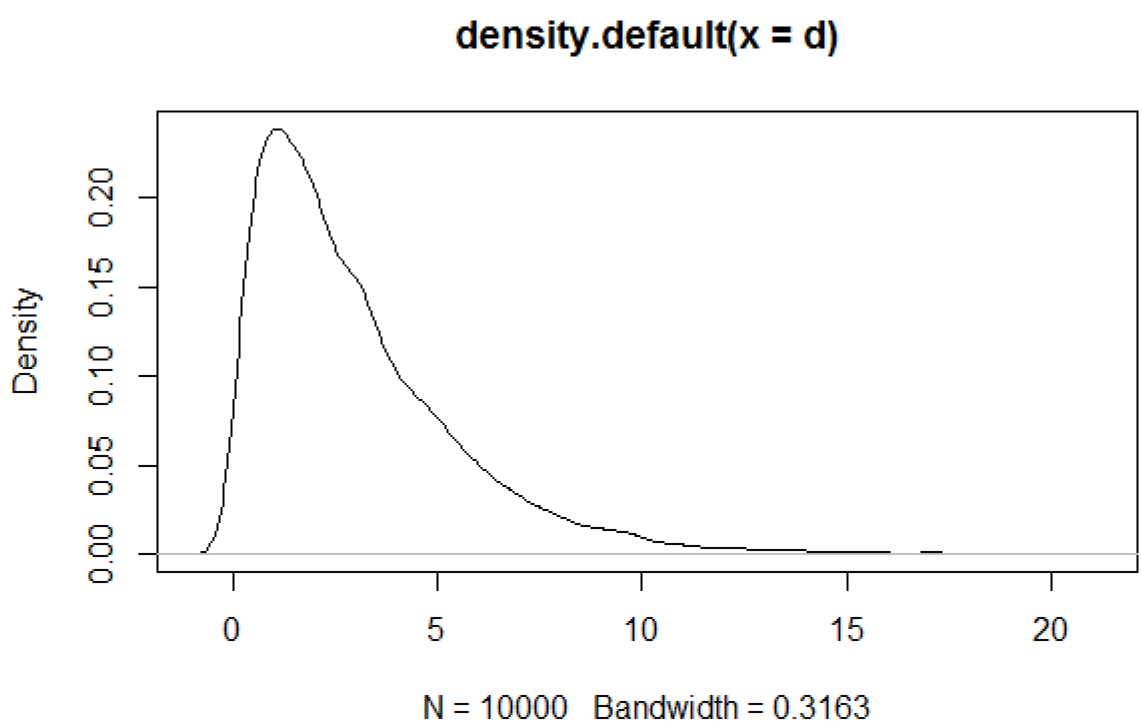
$$\frac{1}{(2\pi)^{\frac{4}{2}} (1/9)^{1/2}} \exp\left\{-\frac{1}{2} V^T \Sigma^{-1} V\right\}$$

```
#####Q3(a)########
library(MASS)
A<-matrix(c(1,2,-1,2,1,-2,-1,3,1,-2,1,3),nrow=4,ncol=3,byrow=TRUE)
A;
mu<-c(1,2,3,-2)
sigma<-A%*%t(A)
y<-mvrnorm(n=1000,mu,sigma);
y.max<-numeric()
for(i in 1:1000)
{
y.max[i]<-max(y[i,])
}
mean(y.max)
var(y.max)
```



```
##########Q3(b)##################
sigma<-A%*%t(A)
rankMatrix(sigma)
d<-rchisq(10000,3)
mean(d)
var(d)
hist(d)
plot(density(d))
```


Histogram of d
Frequency
0
500
1000
2000
0
5
10
15
20
d


density.default(x = d)
Density
0.00
0.05
0.10
0.15
0.20
0
5
10
15
20
N = 10000   Bandwidth = 0.3163