The equations of displacement for this system are generally:

$$m_i \frac{d^2\xi_i}{dt^2} = -k_i\, \xi_{i-1} - (k_i + k_{i+1})\xi_i + k_{i+1}\xi_{i+1}\,, i = 2, \dots, n-1$$

Which only serve for n=2 for said system as they are. For the other two cases, we have

$$m_i \frac{d^2\xi_i}{dt^2} = -(k_i + k_{i+1})\xi_i + k_{i+1}\xi_{i+1}\,, i = 1$$

$$m_i \frac{d^2\xi_i}{dt^2} = -k_i\, \xi_{i-1} - (k_i + k_{i+1})\xi_i\,, i = 3$$

Since $k_i = 1$ for all springs and $m_2 = M, m_{1,3} = 1$, we get

$$\frac{d^2x_1}{dt^2} = -x1 + x_2$$

$$M\frac{d^2x_2}{dt^2} = x_3 - 2x_2 + x_1$$

$$\frac{d^2x_3}{dt^2} = -x_3 + x_2$$

b) The solutions of the displacement equations are $\xi_i(t) = u_i e^{i\omega t}$. Substituting these solutions to the equations derived in a), we get the following system:

$$\begin{pmatrix} \omega^2 - 1 & 1 & 0 \\ 1 & M\omega^2 - 2 & 1 \\ 0 & 1 & \omega^2 - 1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = 0$$

The eigenvalues of which are derived by setting the determinant equal to 0. This way, we obtain the equation

$$(\omega^2 - 1)[(M\omega^2 - 2)(\omega^2 - 1) - 2] = 0$$

The solutions of which are $\omega = 0, \omega = 1\ and\ \omega^2 = 1 + \frac{2}{M}$

c) Plugging the eigenvalues for ω in the system, we get

$$\begin{pmatrix} -1 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = 0, \omega = 0$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & M - 2 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = 0, \omega = 1$$

$$\begin{pmatrix} \frac{2}{M} & 1 & 0 \\ 1 & M & 1 \\ 0 & 1 & \frac{2}{M} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = 0, \omega = 1 + \frac{2}{M}$$

For ω=0, the first and the second row give $u_1 = u_2 = u_3$, but since there is no frequency , the bodies are at rest. For ω=1, the solutions are $u_1 = -u_3, u_2 = 0$. This means that the body B does not move, but the other two oscillate around it, in opposite directions every time. Finally, for $\omega = 1 + \frac{2}{M}$,

$$(1 - \frac{2}{M})u_1 = u_3$$

$$u_2 = -\frac{2}{M} (\frac{1 - \frac{1}{M}}{1 - \frac{2}{M}})\, u_3$$

Where all bodies oscillate, but the direction of the oscillation depends on the mass M of the second body.

- If $M < 1, u_1 and\ u_3$oscillate in the opposite directions, and $u_2$oscillates in the same direction with u_1.
- If $1 > M > 2, u_1 and\ u_3$oscillate in the opposite directions, and $u_2$oscillates in the same direction with u_3.
- If $M > 2, u_1 and\ u_3$oscillate in the same directions, and $u_2$oscillates in the opposite direction with both of them.