ASSESSMENT TASK

BUCKLING OF A PIN-ENDED STRUT

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ABSTRACT

The experiment was performed to examine the buckling phenomenon of the struts. After the completion of experiment results were compared with the theoretical results. Three struts (steel, brass and aluminum) of different length were used in this experiment. All the specimens were pin-end joints. Load was applied on the specimen at the specified locations on the specimen. All the pin-end struts of various materials were tested. Finally the critical load measured was compared with the theoretical results i.e. Euler predictions. It was hard to find out the critical load in the aluminum struts since it has a plastic behavior in the deformation. For the given steel struts the experimental results are close to the Euler predictions.

AIM

The aim of the experiment is to determine, by experimental methods, the imperfections inherent in struts, for up to three materials i.e. steel, brass and aluminum.

INTRODUCTION

Generally in a truss structure member which undergoes in tension was called as a tension member and member which undergoes compression is called as a strut member. Very short columns will fail at the crushing load, but the compression members like struts are not fail entirely due to crushing. These members are considerably long in comparing with the least lateral dimension. Hence these strut members start bending i.e., buckling withe load reaches a certain critical value.

Once a member shows a signs of buckling it will lead to the failure of a member. This load at which the member just buckles is called the buckling load or critical load or crippling load. The buckling load isless than the crushing load. The value of the buckling load is low for long members and relatively high for short members. The value of the buckling load for a given member depends upon the length of the member and the least lateral dimension. When an axially loaded compression member just buckles, it issaid to develop an elastic instability.

Failure of the long columns due to buckling phenomenon was discussed by Euler and his theory on long columns solved so many problems. In his theory he was discussed four cases

- case 1 when both ends of the members are pinned
- case 2 when one end is fixed and the other is free
- case 3 when both ends are fixed
- case 4 when one end is fixed and the other is pinned

Case 1 is considered in the experiment since the both ends of the strut member are pin jointed.

BACKGROUND

Euler analysis of this type of problems have made following assumptions

- Strut material should be homogeneous and it should be linearly elastic i.e. it should obeys Hooke's law.
- Strut should be perfectly straight and should be no imperfections
- Loading should be applied on the ends of the strut and it should be applied at the centroid of the cross section at the ends.

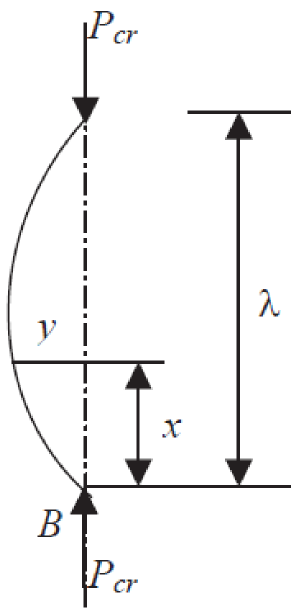

Figure 1: Strut Buckling

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Initially strut will remain in straight for all values of the P, but later at some value i.e. P = P_{cr} strut will buckle.

The least value from the all critical loads i.e. which causes the buckling phenomenon was given by

$P_{cr} = \frac{\pi^2 EI}{L^2}$

Where P_{cr}= Critical or Euler buckling load = P_e(N)

E = Young's modulus ($\frac{N}{m^2}$)

I = Second moment of Inertia (m^4)

L = Length of the strut

Generally for eccentric (e_0) and c_0 type imperfections, the central displacement value i.e. deflection at the middle of the strut is given by below formulae.

For $e_0$type imperfection deflection was calculated

$\delta = e_0[\text{Sec}\ (\frac{\mu L}{2}) - 1]$

Where

δ = Central deflection

e_0= eccentricity of the load application on strut

$\mu = (\frac{P}{EI})^{1/2}$

L = Length of the strut

For $c_0$type imperfection deflection was calculated

$\Delta = c_0[\frac{P}{P_e - P}]$

Where

Δ = Central deflection

P = applied load

P_e = Euler Buckling load

METHODOLOGY

1. The length of each bar is to be noted, together with the cross-sectional dimensions.
2. Each bar is to be loaded in the specified testing machine, under load control, up to a value close to the buckling stage.
3. Results should be noted with accuracy and it should be compared with the theoretical Euler prediction values.
4. Plot should be drawn between load and deflection.
5. Errors should be minimized.

CALCULATIONS & RESULTS

1. Aluminum Strut (e_0 type imperfection)

Specifications of strut

Depth = D = 4.81 mm

Width = B = 19.1 mm

Length = L = 733 mm

Eccentricity taken ($e_{0\ actual}$) = 5.61 mm

Young's modulus = E = 69 x 10^3 MPa = 69 x $10^3 \frac{N}{mm^2}$

Moment of Inertia = I = $\frac{B\,D^3}{12} = \frac{19.1 \text{ x } 4.81^3}{12}$ = 177.12 mm^4

Euler buckling load (theoretical) = $P_{e(theoretical)} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 \text{x } 69000 \text{ x } 177.12}{733^2}$ = 224.67 N

Critical Stress $\sigma_{cr} = \frac{Pe}{A} = \frac{224.67}{4.81 \text{ x } 19.1} = 2.44\ \frac{N}{mm^2}$

Since the load considered should be 75% - 80% of the obtained Euler buckling load value.

$P_{e(theoretical)}$(75%) = 168.502 N

$P_{e(theoretical)}$(80%) = 179.736 N

Table 1: Aluminum strut (e0 type) calculations

Load P (N)	Deflection δ_{actual} (mm)	μ	$\frac{\mu L}{2}$	$\sec(\frac{\mu L}{2})$	$\sec(\frac{\mu L}{2}) - 1$	Deflection $\delta_{theoretical}$(mm)	$e_{experimental}$
15	0.49	0.0011078	0.406	1.088	0.088	0.496	5.53
20	0.76	0.0012792	0.468	1.121	0.121	0.678	6.28
25	1	0.0014302	0.524	1.155	0.155	0.870	6.44
30	1.29	0.0015667	0.574	1.191	0.191	1.071	6.75
35	1.6	0.0016922	0.620	1.228	0.228	1.283	6.99
40	1.85	0.0018090	0.663	1.268	0.268	1.508	6.88
45	2.2	0.0019188	0.703	1.311	0.311	1.745	7.07
50	2.6	0.0202263	0.741	1.355	0.355	1.995	7.30
55	3.02	0.0021213	0.777	1.403	0.403	2.261	7.49
60	3.32	0.0022156	0.812	1.453	0.453	2.543	7.32
70	4.35	0.0023932	0.877	1.564	0.564	3.164	7.71
80	5.47	0.0025584	0.937	1.690	0.690	3.871	7.92
90	6.83	0.0027136	0.994	1.835	0.835	4.685	8.17
100	8.54	0.0028604	1.048	2.003	1.003	5.632	8.50

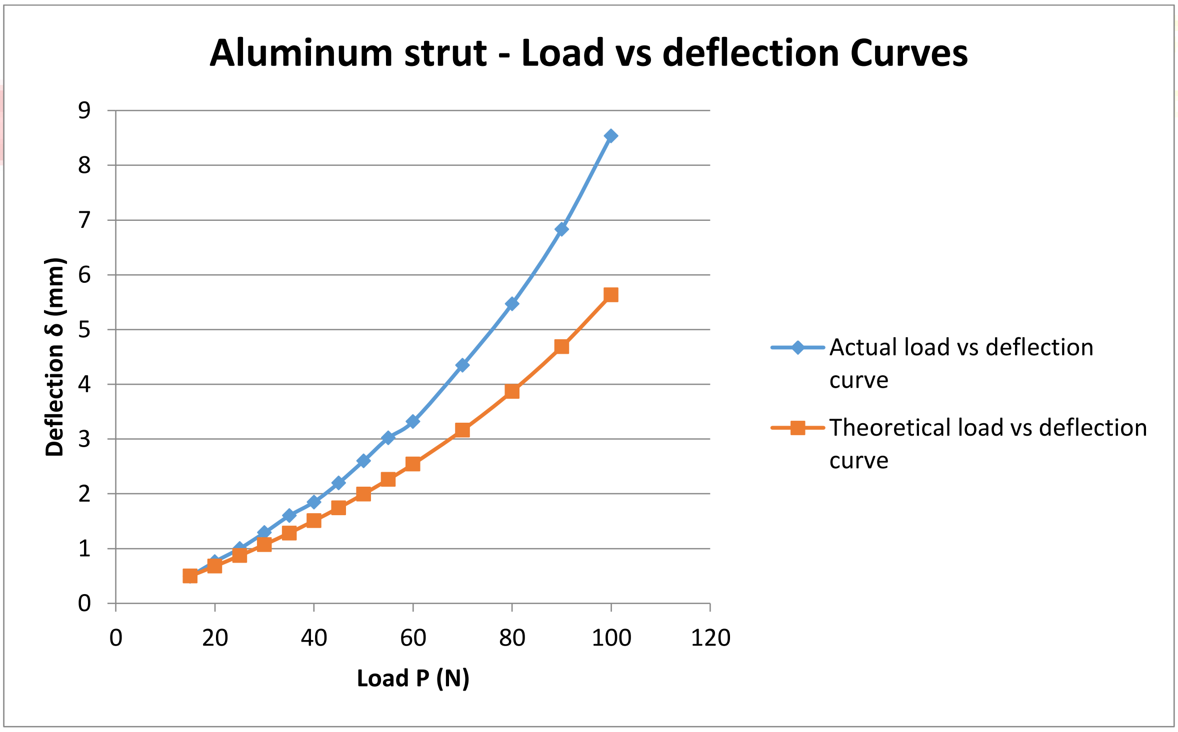

Figure 2: Aluminum load vs deflection plot

2. Steel Strut (e_0 type imperfection)

Specifications of strut

Depth = D = 3.17 mm

Width = B = 19.1 mm

Length = L = 733 mm

Eccentricity taken ($e_{0\ actual}$) = 4.7 mm

Young's modulus = E = 207 x 10^3 MPa = 207 x $10^3 \frac{N}{mm^2}$

Moment of Inertia = I = $\frac{B\,D^3}{12} = \frac{19.1 \times 4.81^3}{12}$ = 50.7026 mm^4

Euler buckling load (theoretical) = $P_{e(theoretical)} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 \times 207000 \times 50.7026}{733^2}$ = 192.9486 N

Critical Stress $\sigma_{cr} = \frac{Pe}{A} = \frac{192.9486}{3.17 \times 19.1} = 3.186 \frac{N}{mm^2}$

Since the load considered should be 75% - 80% of the obtained Euler buckling load value.

$P_{e(theoretical)}$(75%) = 144.711 N

$P_{e(theoretical)}$(80%) = 154.358 N

Table 2: Steel strut (e0 type)calculations

Load P (N)	Deflection δ actual (mm)	μ	$\frac{\mu L}{2}$	sec $(\frac{\mu L}{2})$	sec $(\frac{\mu L}{2})$ - 1	Deflection δ theoretical(mm)	$e_{experimental}$
0	0	0	0	1	0	0	-
15	0.83	0.001195	0.4381	1.1043	0.1043	0.1043	7.95
30	1.79	0.001691	0.6196	1.2283	0.2283	0.2283	7.83
45	3.31	0.002071	0.7588	1.3781	0.3781	0.3781	8.75
60	4.92	0.002391	0.8762	1.5624	0.5624	0.5625	8.74
75	7.85	0.002673	0.9797	1.7945	0.7945	0.7945	9.88
90	11.1	0.002928	1.0732	2.0951	1.0951	1.0952	10.1
105	16.3	0.003163	1.1592	2.4997	1.4996	1.4996	10.86

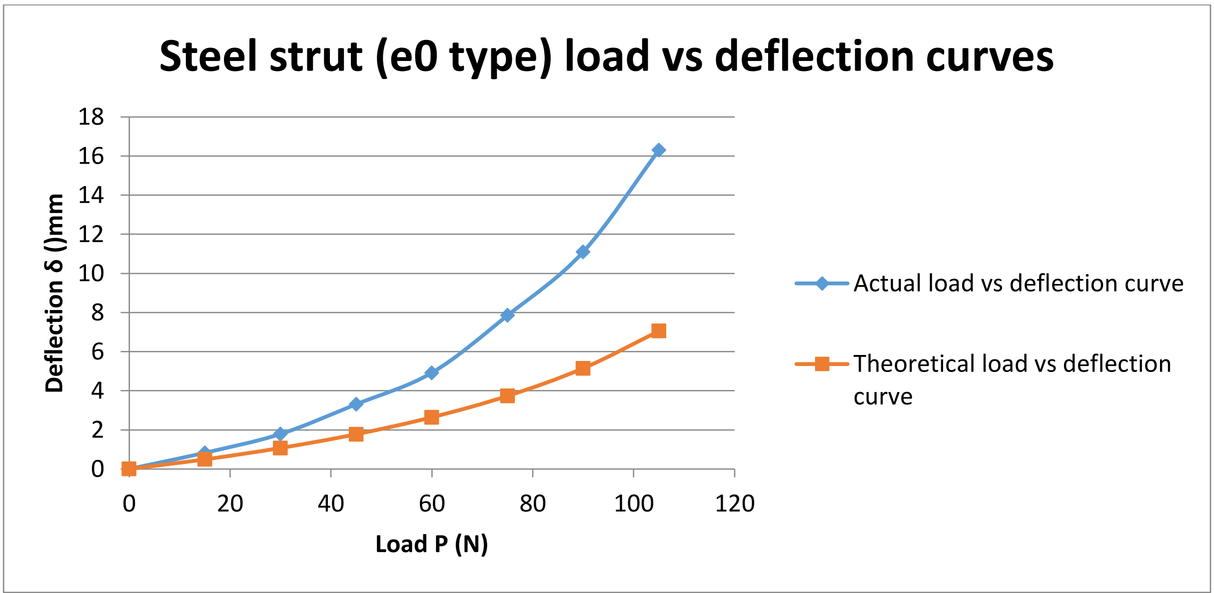

Figure 3: Steel strut (e0 type) load vs deflection plot

3. Brass Strut (e_0 type imperfection)

Specifications of strut

Depth = D = 4.70 mm

Width = B = 18.95 mm

Length = L = 733 mm

Eccentricity taken ($e_{0\ actual}$) = 5.55 mm

Young's modulus = E = 105x 10^3 MPa = 105 x $10^3 \frac{N}{mm^2}$

Moment of Inertia = I = $\frac{B\ D^3}{12} = \frac{18.95 \times 4.70^3}{12}$ = 163.9538 mm^4

Euler buckling load (theoretical) = $P_{e(theoretical)} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 x\ 105000\ x\ 163.9538}{733^2}$ = 316.4843 N

Critical Stress $\sigma_{cr} = \frac{Pe}{A} = \frac{316.4843}{4.70\ x\ 18.95} = 3.553 \frac{N}{mm^2}$

Since the load considered should be 75% - 80% of the obtained Euler buckling load value.

$P_{e(theoretical)}$(75%) = 237.363 N

$P_{e(theoretical)}$(80%) = 253.187 N

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Table 3: Brass strut (e0 type)calculations

Load P (N)	Deflection δ_{actual} (mm)	μ	$\frac{\mu L}{2}$	sec $(\frac{\mu L}{2})$	sec $(\frac{\mu L}{2})$ - 1	Deflection $\delta_{theoretical}$(mm)	$e_{experimental}$
0	0	0	0	1	0	0	-
30	1.09	0.001320	0.4838	1.1296	0.1296	0.719	8.40
60	2.64	0.001866	0.6842	1.2904	0.2904	1.612	9.08
90	4.75	0.002286	0.8379	1.4948	0.4948	2.746	9.59
120	8.08	0.002640	0.9676	1.7628	0.7628	4.233	10.59
150	13.76	0.002951	1.0818	2.1290	0.1290	6.265	12.18
180	24.72	0.003233	1.1850	2.6581	0.6581	9.203	14.90

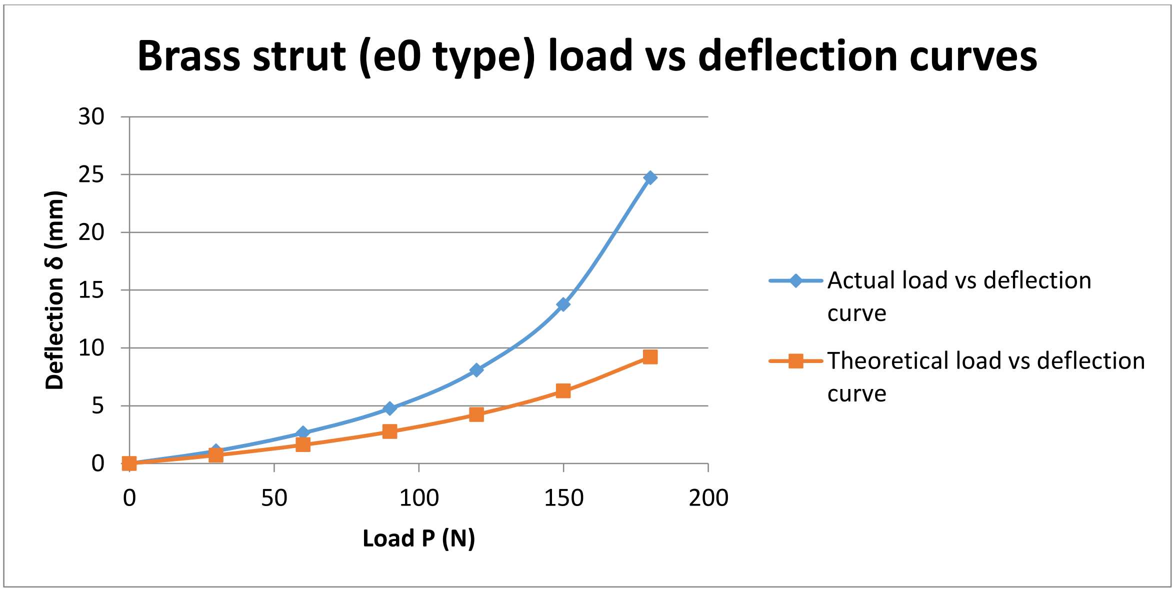

Figure 4: Brass strut (e0 type) load vs deflection plot

4. Brass Strut (c_0 type imperfection)

Specifications of strut

Depth = D = 3.10 mm

Width = B = 19.20mm

Length = L = 702.2 mm

Coefficient of edge rigidity taken ($C_{0\ actual}$) = 1.5 mm

Young's modulus = E = 105x 10^3 MPa = 105 x $10^3 \frac{N}{mm^2}$

Moment of Inertia = I = $\frac{B\,D^3}{12} = \frac{19.20 \text{ x } 3.10^3}{12}$ = 47.6656 mm^4

Euler buckling load (theoretical) = $P_{e(theoretical)} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 \text{x } 105000 \text{ x } 47.6656}{702.2^2}$ = 100.2587 N

Critical Stress $\sigma_{cr} = \frac{Pe}{A} = \frac{100.2587}{3.10 \text{ x } 19.20} = 1.684 \frac{N}{mm^2}$

Since the load considered should be 75% - 80% of the obtained Euler buckling load value.

$P_{e(theoretical)}$(75%) = 75.1940 N

$P_{e(theoretical)}$(80%) = 80.2069 N

Table 4: Brass strut (c0 type)calculations

Load P (N)	Deflection Δ_{actual} (mm)	$\frac{P}{p - p_e}$	Deflection $\Delta_{theoretical}$(mm)	$C_{0experimental}$
0	0	0	0	-
10	0.25	0.1107	0.166	2.256

20	0.52	0.2492	0.373	2.086
30	1	0.4269	0.640	2.341
35	1.26	0.5363	0.804	2.349
40	1.41	0.6638	0.995	2.124
45	1.8	0.8143	1.221	2.210
50	1.98	0.9948	1.492	1.990
55	2.43	1.2152	1.822	1.999
60	3.01	1.4903	2.235	2.019

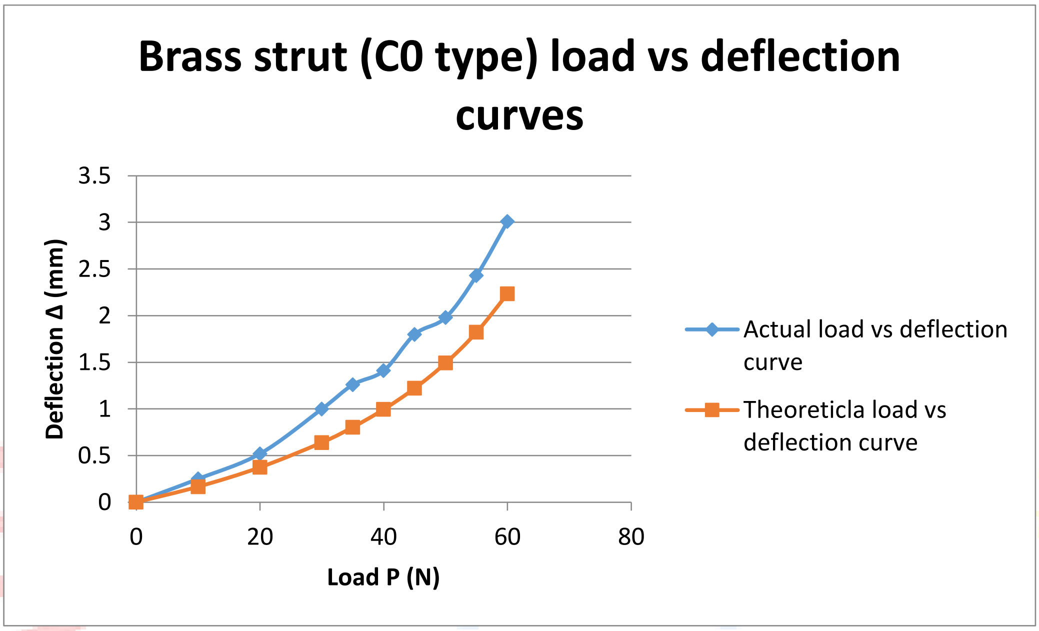

Figure 5: Brass strut (co type) load vs deflection plot

5. Steel Strut (c_0 type imperfection)

Specifications of strut

Depth = D = 2.90 mm

Width = B = 20.00 mm

Length = L = 504.00 mm

Coefficient of edge rigidity taken ($C_{0\ actual}$) = 1.0 mm

Young's modulus = E = 207x 10^3 MPa = 207 x $10^3 \frac{N}{mm^2}$

Moment of Inertia = I = $\frac{B\,D^3}{12} = \frac{20.00 \times 2.90^3}{12}$ = 40.6483 mm^4

Euler buckling load (theoretical) = $P_{e(theoretical)} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 x\ 207000 \times 40.6483}{504.00^2}$ = 327.191 N

Critical Stress $\sigma_{cr} = \frac{Pe}{A} = \frac{327.191}{2.90 \times 20.00} = 5.64 \frac{N}{mm^2}$

Since the load considered should be 75% - 80% of the obtained Euler buckling load value.

$P_{e(theoretical)}(75\%) = 245.393$ N

$P_{e(theoretical)}(80\%) = 261.752$ N

Table 5: Steel strut (c0 type) calculations

Load P (N)	Deflection Δ actual (mm)	$\frac{\Delta}{P}$	$\frac{P}{p - p_e}$	Deflection Δ theoretical(mm)	$C_{0experimental}$
0	0	0	0	0	-
30	0.04	0.0013	0.1009	0.1009	0.3962
60	0.12	0.0020	0.2245	0.2245	0.5343
90	0.24	0.0026	0.3794	0.3794	0.6325
120	0.45	0.0037	0.5791	0.5791	0.7769
150	0.73	0.0048	0.8465	0.8465	0.8623
180	1.04	0.0058	1.2229	1.2229	0.8504
190	1.24	0.0065	1.3849	1.3849	0.8953
200	1.38	0.0069	1.5724	1.5724	0.8776
210	1.72	0.0082	1.7919	1.7919	0.9598
220	1.82	0.0083	2.052	2.052	0.8867

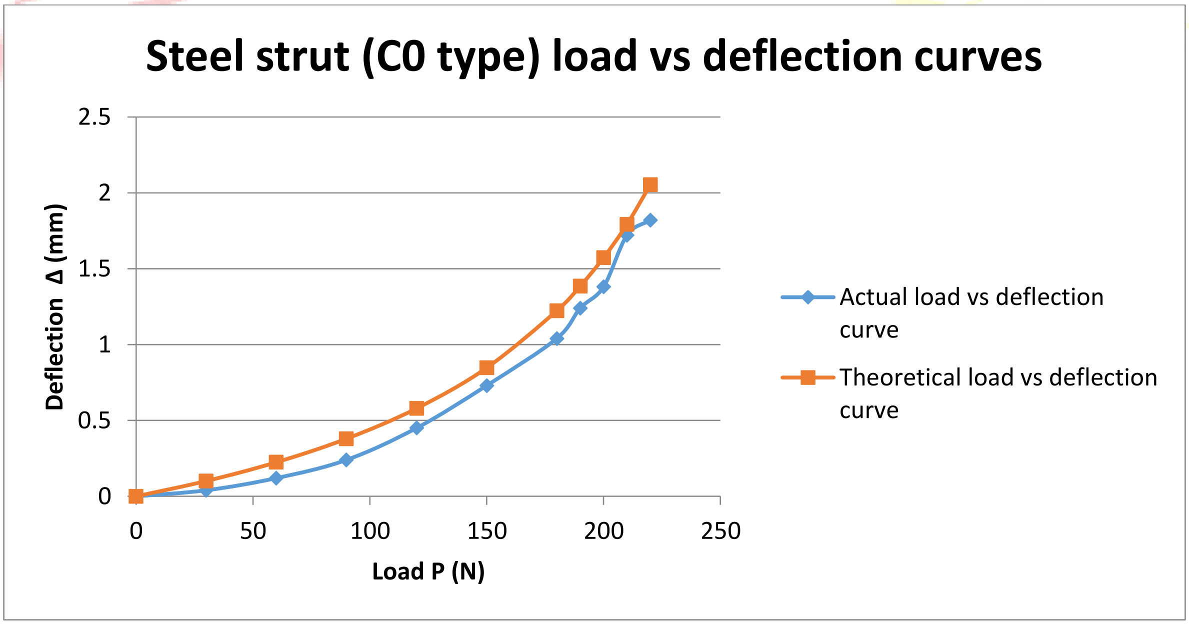

Figure 6: Steels strut (c0 type) load vs deflection plots

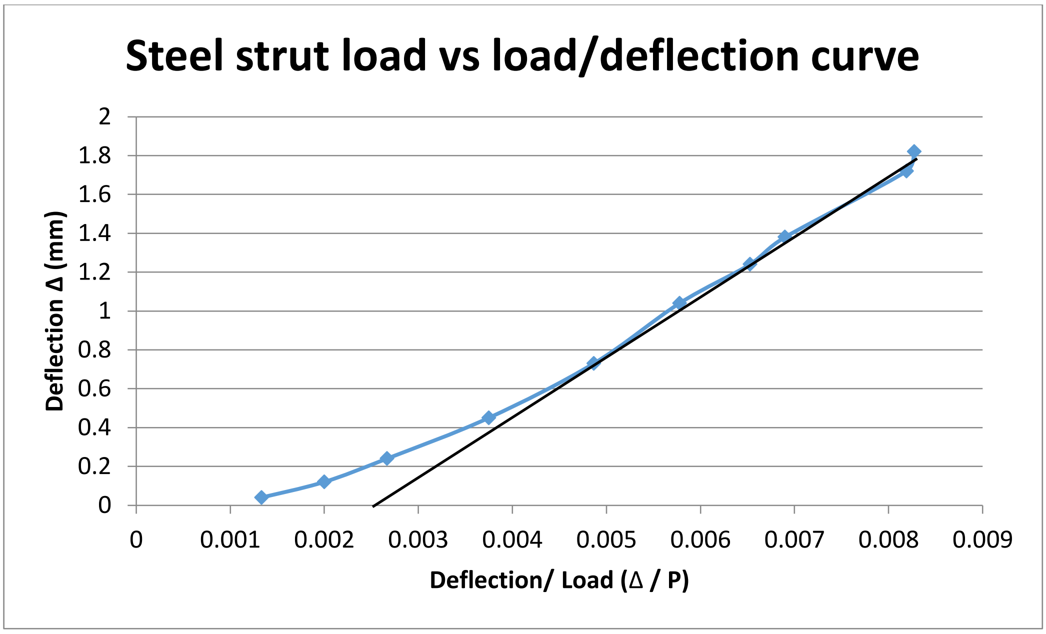

Figure 7: Steel strut load vs load/deflection plots

Slope = $\frac{1.82-0.00}{0.008273-0.00275}$ = 329.53 N = $P_{cr}$from the experimental calculations

DISCUSSION

When the experiment was carried out human errors are not escapable. These errors can be minimized by following the specified precautions as mentioned. Even though these errors are minimized machinery errors were also causes the errors in calculations. That's why the errors are incorporated in the results obtained. Error may increases due to machinery defects, impurities in materials, imperfection in fixing of specimen etc. Aluminum strut theoretical deflection values are nearly matches the experimental calculations but steel and brass struts experimental results were not matching the theoretical results. It may be due to the initial imperfection, fixity imperfections, machinery defects and some human errors. But from the curves the Euler prediction was almost matches with the experimental calculations. Southwell method also used to evaluate the buckling phenomenon of struts.

CONCLUSION

The modulus of elasticity or young's modulus for various materials was discussed. Pin ended Aluminum, steel and brass struts were tested. Errors due to imperfections were noted. Idealized conditions and assumptions were assumed when the experiment was conducted. No hysteresis errors were identified during the experiment. All the struts were tested and the results were compared with the Euler theoretical predictions.

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