1. The aerodynamic lift of the wing is described by the distributed load of

\[ w = -300\sqrt{1 - 0.04x^2} \text{ N/m}. \]

The mass of the wing is 27 Kg, and its center of mass is located 2 m from the wing root \( R \).

a. Determine the magnitude of the force and the moment about \( R \) exerted by the lift of the wing.

The magnitude of the force \( F \) is equal to the area between the curve of

\[ w = -300\sqrt{1 - 0.04x^2} \text{ and x-axis. Thus} \]

\[ F = -\int_0^5 300\sqrt{1 - (0.2x)^2} \, dx = -300\int_0^2 \sqrt{1 - y^2} \, dy = -1500\int_0^2 \sqrt{1 - y^2} \, dy \]

\[ = -1500\left[ \frac{1}{2} \left( y\sqrt{1 - y^2} + 1^2 \cdot \arcsin\left(\frac{y}{1}\right) \right) \right]^2_0 = -1500\frac{1}{2}(\arcsin(1) - \arcsin(0)) = -1500\frac{\pi}{4} = -375\pi \]

The magnitude of the Moment \( M \) is equal to the following

\[ M = \int_0^5 x \cdot w \, dx = -300\int_0^5 x \cdot \sqrt{1 - (0.2x)^2} \, dx = -7500\int_0^2 y \cdot \sqrt{1 - y^2} \, dy = -7500\int_0^{\pi/2} \sin t \cos^2 t \, dt \]

\[ = -7500\int_0^{\pi/2} \left( \sin t - \sin^3 t \right) dt = -7500\left( -\cos t + \frac{2}{3} \cos^3 t \right)^{\pi/2}_0 = -7500\left( 1 - \frac{2}{3} \right) = \frac{7500}{3} = -2500 \text{N} \cdot \text{m} \]

The location of the center of lift force is as follows

\[ x = \frac{M}{F} = \frac{2500}{375\pi} = 2.122 \text{m} \]

b. Determine the reactions on the wing at \( R \).

Reaction force: \(- (F + mg) = 375\pi - 27 \cdot 9.8 = 913.5 \text{N} \)

Reaction moment: \(-M - (M + mg \cdot 2) = 2500 - 27 \cdot 9.8 \cdot 2 = 2500 - 529.2 = 1970.8 \text{N} \cdot \text{m} \)
2. Given a beam under distributed loads, develop a MATLAB program to determine the equivalent resultant force of the distributed loading and its location, measure from point A. (Hint: type 'help quad' in MATLAB command window, and read the description) Use the values, $C_1$=5, $C_2$=16, $a$ = 3 and $b$ = 1.

```matlab
rc = 15;
a = 1;
b = 1;
p = acos(-1);

% distribution loads function
W = @(x)sqrt((rc*x + sqrt(2*r*c2*x^2 + 4))); % distribution moment function
M = @(x)W(x)*sqrt(1 - 0.5*x/a); % distribution torque function
F = @(x)3*W(x)/a;
hold on;
p = -300:50:300;
plot([0,a],[0,0],'-k','linewidth',5);
plot([a,b],[0,0],'-b','linewidth',5);

X = 1.681
Y = 10.06

disp('Equivalent resultant force in a = 10.0556')
disp('The location of center = 1.6814')
disp('Equivalent resultant force in b = 4.7639')
disp('The location of center = 3.5104')
```
3. Consider an airfoil with chord length $c$ and running distance $x$ measured along the chord. The leading edge is located at $(x/c) = 0$ and the trailing edge at $(x/c) = 1$. The pressure coefficient variations over the upper and lower surfaces are given, respectively, as

$$C_{p,u} = 1 - 300 \left( \frac{x}{c} \right)^2 \quad \text{for} \quad 0 \leq \left( \frac{x}{c} \right) \leq 0.1$$

$$C_{p,l} = -2.2277 + 2.2777 \left( \frac{x}{c} \right) \quad \text{for} \quad 0.1 \leq \left( \frac{x}{c} \right) \leq 1.0$$

$$C_{p,l} = 1 - 0.95 \left( \frac{x}{c} \right) \quad \text{for} \quad 0 \leq \left( \frac{x}{c} \right) \leq 1.0$$

a. Using MATLAB, plot the pressure distribution

b. Calculate the normal force coefficient

$$C_p = C_{pl} - C_{pu} = \begin{cases} 
-0.95 \left( \frac{x}{c} \right) + 300 \left( \frac{x}{c} \right)^2 & \text{for} \quad 0 \leq \left( \frac{x}{c} \right) \leq 0.1 \\
3.2277 - 3.2277 \left( \frac{x}{c} \right) & \text{for} \quad 0.1 \leq \left( \frac{x}{c} \right) \leq 1.0
\end{cases}$$
c. Calculate the location of center of pressure

\[ X_c = 0.378 \]
\[ F = 1.402 \]

4. The figure shown below represents the pressure distribution on a NACA2412 airfoil section at \( \alpha = 5 \) degree. For a quick estimation of lift coefficient, these curves are approximated as the dotted line as shown.

a. Find the estimated value of lift coefficient

\[
C_{pu} = \begin{cases} 
-2 + 7 \left( \frac{x}{c} \right) & \text{for } 0 \leq \left( \frac{x}{c} \right) \leq 0.1 \\
-1.4444 + 1.4444 \left( \frac{x}{c} \right) & \text{for } 0.1 \leq \left( \frac{x}{c} \right) \leq 1 
\end{cases}
\]
\[
C_{pl} = \begin{cases}
1 - 7.5 \left( \frac{x}{c} \right) & \text{for } 0 \leq \left( \frac{x}{c} \right) \leq 0.1 \\
0.2777 - 0.2777 \left( \frac{x}{c} \right) & \text{for } 0.1 \leq \left( \frac{x}{c} \right) \leq 1
\end{cases}
\]

\[
C_p = \begin{cases}
3 - 14.5 \left( \frac{x}{c} \right) & \text{for } 0 \leq \left( \frac{x}{c} \right) \leq 0.1 \\
1.7222 - 1.7222 \left( \frac{x}{c} \right) & \text{for } 0.1 \leq \left( \frac{x}{c} \right) \leq 1
\end{cases}
\]

b. Find the estimated location of center of lift

Force: 0.9250

Location of the center: 0.31261

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